Iterative MMSE cooperative localization with incomplete pair-wise range measurements

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ABSTRACT

We propose and study an iterative minimum-mean-square-error (MMSE) cooperative localization algorithm, which achieves better root-mean-square-error (RMSE) performance than existing classical estimators. Using the received signal strength (RSS) measurements, we first derive the formulas for estimating the coordinates of position-unknown nodes. Then, we investigate the practical solutions to calculate the complicated multiple integrals involved in the formulas and propose an adaptive and iterative algorithm to circumvent the intense computation burden incurred by the numerical multiple integral computation methods. We further study the proposed MMSE cooperative localization algorithm in the scenario where pair-wise range measurements are incomplete, that is, pair-wise range measurements between certain pairs of nodes are missing. It is observed that the performance degrades at a slower speed than the reduction of the available range measurements. In other words, not much performance degradation is caused by comparatively large number of missing measurements. Therefore, we can improve the efficiency of the iterative MMSE algorithm by intentionally throwing away certain pair-wise range measurements.

Keywords: cooperative localization, MMSE, received signal strength (RSS), incomplete measurements

1. INTRODUCTION

The global positioning system (GPS) signal is not always accessible for localization, such as inside a building or for a low budget network. This has motivated extensive study on non-GPS localization techniques. Cooperative localization, where the position of any node is estimated based on all pair-wise measurements, is drawing more and more attention due to its superior performance over non-cooperative localization.

The most popular cooperative localization techniques include the multidimensional scaling (MDS)[1] method and the maximum likelihood estimator (MLE) method.[2 5] In MDS localization method, the coordinates of all unknown nodes are solved as the least square solutions to an overdetermined equation set formed using all distances between each pair of nodes. The singular value decomposition (SVD) is commonly used to obtain the MDS solutions. Details of solving MDS via SVD can be found in the previous work.[6] For MLE method, extensive study has shown that MLE is an asymptotic unbiased estimator that is asymptotically efficient since its variance is close to the Cramér-Rao bound (CRB). However, MLE solution has to be obtained iteratively and since the optimization problem involved is not a convex optimization problem, it is not guaranteed that global maximizer can be achieved starting with any initial estimation.[7] Both MDS and MLE methods, as well as other existing cooperative localization techniques, are based on classical estimation where no priori distribution of the true positions of nodes is considered and their performance in terms of root-mean-square-error (RMSE) is bounded by the Cramér-Rao bound (CRB).

According to the general discussion on relationship between priori and posterior knowledge in Bayesian estimation in,[8] it is expected that better estimators can be obtained via Bayesian estimation, by treating the true position of nodes as random unknown parameters distributed according to a priori probability density function (PDF). With this motivation, we propose and study cooperative localization based on minimum mean square error (MMSE) estimation, whose performance is verified to be much better than that of MDS and MLE. In fact, it can even break the CRB.

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There are four types of measurements[9] for position estimation, which are the received signal strength indicator (RSSI), the angle of arrival (AOA), the time of arrival (TOA) and the time-distance of arrival (TDOA). RSSI and TOA are mostly used in practical applications. TOA is not always available, therefore throughout this work, like in,[10] we adopt RSSI, or equivalently the received power strength, as measurements.

Complicated multiple integrals are involved in MMSE formulas. Direct computation of these multiple integrals have to be done via numerical methods and the computation burden gets extremely high as the number of nodes gets large. To solve this issue, we propose to carry out MMSE estimation in a successively iterative pattern to avoid multiple integrals of large number of folds. Details are given later. Numerical results are shown to verify that such an iterative MMSE reduce the complexity without losing superior performance of original MMSE.

There exist certain situations where not all pair-wise measurements can be obtained. In other words, we may have incomplete measurements. Incomplete measurements can be caused by obstacles or jamming between certain pairs of nodes so that the received signal strength is not detectable. Or, some RSS measurements are missing due to the long distance between the corresponding pairs of nodes. We examine how the proposed algorithm works in case of incomplete measurements by disregarding pair-wise measurements between some pairs of nodes, or in short words, by disregarding some paths. We observed that a large amount of missing measurements, while consuming much less computation operations, only cause a smaller amount of performance degradation. Motivated by this observation, we propose to improve the computation efficiency of the iterative MMSE algorithm by intentionally not using certain measurements.

The remaining of this paper is organized as follows. After introducing the system model in Section 2, the proposed MMSE cooperative localization algorithm and its iterative solution are presented in Section 3. Then in Section 4, incomplete measurements are considered for further study of the proposed iterative MMSE algorithm. The superior performance of the proposed iterative MMSE estimator with complete or incomplete measurements is verified by the numerical results with some analysis presented in Section 5. Conclusion of this paper is addressed in Section 6.

2. SYSTEM MODEL

Anchor nodes, whose positions are known in advance, are needed to estimate the coordinates of position-unknown nodes, which are called *unknown nodes* for brevity.

Consider a wireless network of N unknown nodes and M anchor nodes. The position for any unknown node i, $1 \leq i \leq N$, is described by its coordinates (x_i, y_i). The node $j = N+1, \cdots, M$ refers to one of M anchor nodes. All nodes transmit at a fixed known power level. The received signal power between any pair of nodes is observed to estimate the coordinates of unknown nodes.

Let P_{ij} denote the received power strength at the node i from the node j, whose distance is denoted as d_{ij}. As in,[11] we adopt the classical log-normal distribution[12] for P_{ij}. Thus, we have

$$P_{ij}(dB) \sim \mathcal{N}\left(\bar{P}_{ij}(dB), \sigma_{dB}^2\right) \tag{1}$$

where $\bar{P}_{ij}(dB)$ is the expectation corresponding to the specific d_{ij} and the variance σ_{dB}^2 keeps the same for any distance. Suppose the average received power strength at distance d_0 is $P_0(dB)$. P_0 and d_0 are called the reference power and distance respectively. According to,[12] we have

$$\bar{P}_{ij}(dB) = P_0(dB) - 10 n_p log_{10}\left(\frac{d_{ij}}{d_0}\right), \tag{2}$$

where n_p is the *path loss exponent*.

3. ITERATIVE MMSE COOPERATIVE LOCALIZATION

3.1 MMSE Cooperative Localization

Let $\boldsymbol{\theta}_i = (x_i, y_i)$ and then the unknown vector parameter to be estimated can be expressed as $\boldsymbol{\theta} = (\boldsymbol{\theta}_1, \cdots, \boldsymbol{\theta}_N)$. As in,[13] the priori distribution for unknown nodes is assumed to be independent uniform distribution. Suppose the node i may appear within a rectangular box centered at (O_{ix}, O_{iy}) of $2A_i$ long along x-axis and $2B_i$ long along y-axis. Then, for $x_i \in (O_{ix} - A_i, O_{ix} + A_i), y_i \in (O_{iy} - B_i, O_{iy} + B_i)$, $1 \le i \le N$, the priori PDF is expressed as

$$f(\boldsymbol{\theta}) = f(x_1, y_1, \cdots, x_N, y_N) = \prod_{i=1}^{N} \frac{1}{2A_i} \prod_{i=1}^{N} \frac{1}{2B_i}. \tag{3}$$

With the independent uniform priori distribution (3) and the log-normal distribution for power degradation (1), the MMSE cooperative position estimator for node i, $1 \le i \le N$, can be derived as

$$\begin{aligned} \hat{x}_{i,\mathrm{MMSE}} &= \frac{\int\limits_{S_N} \cdots \int\limits_{S_1} x_i \prod\limits_{i=1}^{N} \prod\limits_{j=i+1}^{N+M} \exp\left(-\frac{\alpha}{8} \ln^2 \frac{d_{ij}^2}{\hat{d}_{ij}^2}\right) d\boldsymbol{\theta}_1 \cdots d\boldsymbol{\theta}_N}{\int\limits_{S_N} \cdots \int\limits_{S_1} \prod\limits_{i=1}^{N} \prod\limits_{j=i+1}^{N+M} \exp\left(-\frac{\alpha}{8} \ln^2 \frac{d_{ij}^2}{\hat{d}_{ij}^2}\right) d\boldsymbol{\theta}_1 \cdots d\boldsymbol{\theta}_N} \\ \hat{y}_{i,\mathrm{MMSE}} &= \frac{\int\limits_{S_N} \cdots \int\limits_{S_1} y_i \prod\limits_{i=1}^{N} \prod\limits_{j=i+1}^{N+M} \exp\left(-\frac{\alpha}{8} \ln^2 \frac{d_{ij}^2}{\hat{d}_{ij}^2}\right) d\boldsymbol{\theta}_1 \cdots d\boldsymbol{\theta}_N}{\int\limits_{S_N} \cdots \int\limits_{S_1} \prod\limits_{i=1}^{N} \prod\limits_{j=i+1}^{N+M} \exp\left(-\frac{\alpha}{8} \ln^2 \frac{d_{ij}^2}{\hat{d}_{ij}^2}\right) d\boldsymbol{\theta}_1 \cdots d\boldsymbol{\theta}_N} \end{aligned} \tag{4}$$

where ln is the natural logarithm and

$$\int \cdot \, d\boldsymbol{\theta}_i \equiv \int \int \cdot \, dx_i dy_i. \tag{5}$$

S_i is the integral region for $\boldsymbol{\theta}_i = (x_i, y_i)$, expressed as

$$S_i = \left\{ (x_i, y_i) \middle| \begin{array}{l} x_i \in (O_{ix} - A_i, O_{ix} + A_i) \\ y_i \in (O_{iy} - B_i, O_{iy} + B_i) \end{array} \right\}. \tag{6}$$

$$\alpha = \left(\frac{10 n_p}{\sigma_{dB} \ln 10} \right)^2, \tag{7}$$

and

$$\hat{d}_{ij} = d_0 \left(\frac{P_0}{P_{ij}} \right)^{1/n_p}. \tag{8}$$

Obviously, $d_{ij}^2 = (x_i - x_j)^2 + (y_i - y_j)^2$.

For computer simulations, since P_{ij} contributes to MMSE cooperative position estimation (4) only through $\hat{d}_{ij}$, once (x_i, y_i), $i = 1, \cdots, N$ are randomly generated, $\hat{d}_{ij}$ can be obtained according to

$$\hat{d}_{ij} = d_{ij} 10^{\frac{\sigma_{dB}}{10 n_p} G}. \tag{9}$$

where $G \sim \mathcal{N}(0, 1)$ is a standard Gaussian random variable. This practice is also used in.[2,3,5,14]

The major steps of derivation for (4) are given below.

Let $\mathbf{P}$ represent the collection of all measured RSS between each pair of N unknown nodes and between each unknown node and the M anchor nodes. Then, $\mathbf{P}$ can be mathematically expressed as

$$\mathbf{P} = (P_{ij} | 1 \le i \le N, i+1 \le j \le N+M), \tag{10}$$

with the node j with $j = N+1, \cdots, M$ referring to one of the anchor nodes.

According to the well established theory that the conditional mean that is conditioned on the given observation sample minimizes the MSE among all estimators, we can obtain the MMSE coordinates estimator $(\hat{x}_{i,\mathrm{MMSE}}, \hat{y}_{i,\mathrm{MMSE}})$ expressed as

$$\begin{array}{l} \hat{x}_{i,\mathrm{MMSE}} = \mathsf{E}[x_i | \mathbf{P}] \\ \hat{y}_{i,\mathrm{MMSE}} = \mathsf{E}[y_i | \mathbf{P}] \end{array}, \quad i = 1, \cdots, N. \tag{11}$$

To obtain the specific expressions (4), the posterior PDF of $\boldsymbol{\theta}$ given the observation $\mathbf{P}$, which is denoted as $f(\boldsymbol{\theta}|\mathbf{P})$, is needed. According to the independent uniform priori distribution (3) and the log-normal distribution for power degradation (1), it can be derived that

$$f(\boldsymbol{\theta}|\mathbf{P}) = \frac{\prod_{i=1}^{N}\prod_{j=i+1}^{N+M} \exp\left[-\frac{\alpha}{8}\ln^2\frac{d_{ij}^2}{\tilde{d}_{ij}^2}\right]}{\int\limits_{S_N}\cdots\int\limits_{S_1}\prod\limits_{i=1}^{N}\prod\limits_{j=i+1}^{N+M}\exp\left(-\frac{\alpha}{8}\ln^2\frac{d_{ij}^2}{\tilde{d}_{ij}^2}\right)d\boldsymbol{\theta}_1\cdots d\boldsymbol{\theta}_N}. \tag{12}$$

Plugging (12) into (11), we obtain the MMSE coordinates estimator as given by (4).

Without loss of generality, from now on, we focus on the simplest special case of the priori PDF (3) where all unknown nodes take position independently and uniformly within the same square area of unit length centered at $(1/2, 1/2)$, that is, $(O_{ix}, O_{iy}) = (1/2, 1/2)$ and $A_i = B_i = 1/2$ for any i. There is one anchor node at each of the four corners. This model is also employed in.[4,14] Please note that all coordinates share the same unit, therefore it is unnecessary to assign a specific unit to the coordinates.

The multiple integrals in MMSE formula (4) are complicated and have no close form solution. Therefore, we have to resort to numerical methods, such as Simpon quadrature and Monte Carlo methods[15] to compute these integrals. However, these methods can only solve small size networks ($N \leq 3$), because the complexity increases exponentially with the number of unknown nodes. To solve this issue, we propose an iterative MMSE as presented in the next subsection.

3.2 Iterative MMSE Algorithm

In iterative MMSE, the MMSE formulas (4) for the special case $N = 1$ is repeatedly applied with adaptively adjusted priori PDF. Therefore, using iterative MMSE, we only need to deal with two fold multiple integrals and greatly reduce the computation burden. This allows the iterative MMSE to be applied to even very large networks with $N = 30$ or $N = 50$.

The proposed iterative MMSE algorithm is carried out as follows.

1. *Initial Estimation*: The coordinates of each unknown node are estimated solely based on the anchor nodes by ignoring all the other unknown nodes. That is, the initial estimated coordinates $(\hat{x}_i^{(0)}, \hat{y}_i^{(0)})$ for $i = 1, \cdots, N$ are obtained using the MMSE formulas (4) with $N = 1$ and $M = 4$.

2. *Repeat*: At the n-th iteration, for each unknown node i, regard all the other unknown nodes as the temporary anchor nodes whose previously estimated coordinates are treated as their true positions, and implement MMSE estimation with an adaptively adjusted virtual priori PDF. This virtual priori PDF is still uniform distribution within a square area. The previously estimated coordinates of the current node is taken as the center and the edge length is adaptively adjusted for different iterations. In mathematics, $(\hat{x}_i^{(n)}, \hat{y}_i^{(n)})$ are obtained using the MMSE formulas (4) with $N = 1$ and $M = N + M - 1$. For temporary anchor nodes $j = 1, \cdots, i-1$, $(x_j, y_j) = (\hat{x}_j^{(n)}, \hat{y}_j^{(n)})$ and for other temporary anchor nodes $j = i+1, \cdots, N$, $(x_j, y_j) = (\hat{x}_j^{(n-1)}, \hat{y}_j^{(n-1)})$. For the integral region S_i in (6), $(O_{ix}, O_{iy}) = (\hat{x}_i^{(n-1)}, \hat{y}_i^{(n-1)})$.

The edge length $A_i = B_i$ of the square is referred to as *step size.* How to select step size for different iterations is crucial to our algorithm. And it is a very delicate job. If choosing a big size for next iteration of estimation, we speed up the iteration but lose the accuracy. And if small step is used, the convergence speed is slowed down and higher accuracy is not guaranteed to be achieved either. An empirical proper step size is the reciprocal of iteration numbers for the first several iterations and keeps the reciprocal of root square of number of unknown nodes, i.e. $1/\sqrt{N}$, for the remaining iterations. For example, when $N = 25$, the step sizes from the first iteration are 1/1,1/2,1/3,1/4,1/5,1/5... of one unit.

One of our important findings is that the number of needed iterations can be determined as a function of the number of unknown nodes. Suppose the number of unknown nodes is N. Empirically, the iteration number is $3\sqrt{N}$. For example, when $N = 25$, $3\sqrt{25} = 15$ iterations need to be run. The reason is that further iteration has marginal improvement after $3\sqrt{N}$ iterations.

Intuitive interpretation of the superior performance of proposed iterative MMSE is shown as follows. Every time we obtain the estimated coordinates, we have gained more information about the true positions and it is reasonable to believe that the true positions are more likely to be close to the estimated coordinates. By choosing the area of the priori PDF as centered at the estimated position and with reduced edge length, the mean of the priori PDF gets closer to the true position. It is well known that MMSE produce the best performance when the true value is close to the mean, therefore such an iterative MMSE gives very good performance.

4. ITERATIVE MMSE ALGORITHM WITH INCOMPLETE MEASUREMENTS

It may happen that there exists an obstacle between a pair of nodes and the pair-wise RSS between this pair of nodes is not detectable and thus the coordinates have to be estimated using incomplete observed data. The situation of incomplete pair-wise measurement may also be caused by the long distance between a pair of nodes. How the proposed algorithm works with incomplete measurements can be examined via simulations by disregarding certain paths. The paths that are not disregarded are called available paths, corresponding to available pair-wise measurements that can be used for estimation. To have different causes for incomplete data into consideration, we randomly choose the disregarded paths.

While it is expected that incomplete measurements cause performance degradation, it is also noted that fewer available paths need less computation. In fact, the amount of computation for (4) is approximately proportional to the number of exponential items in the function to be integrated. This number is determined by the number of available paths between the currently estimated node and other nodes. Thus, the total amount of computation for obtaining the estimated coordinates of all unknown nodes can be measured by the total number of available paths between any two nodes.

Let L_t denote the total number of paths for a wireless network of N nodes, and let L_a denote the available paths. Then *available paths ratio* is defined as $\frac{L_a}{L_t}$, which indicates how much percentage of all paths can be used. This ratio also indicates how much computation is needed compared with the total amount of computation when all paths are involved. Let RMSE(l) denote the RMSE obtained when l paths are available. Obviously, the performance when all paths available is described by RMSE(L_t) and the performance when some paths are missing and L_a paths are available is described by RMSE(L_a). Since larger RMSE indicates worse performance, we describe the performance degradation by *performance ratio*, which is defined as $\frac{\mathrm{RMSE}(L_t)}{\mathrm{RMSE}(L_a)}$. The best case is when $L_a = L_t$. In this best case, the performance ratio is 1 and the performance degradation is zero. Thus, larger performance ratio means less performance degradation.

The numerical results shown in the next section indicates that we can improve the computational efficiency of the iterative MMSE algorithm by intentionally excluding certain paths.

5. NUMERICAL RESULTS AND ANALYSIS

As explained in Section 3, only two-fold multiple integrals need to be calculated when using iterative MMSE. For up to three fold multiple integrals, we can use both the Simpon quadrature method and the Monte Carlo method. Since the Simpon method is more accurate, we use the Simpon method in iterative MMSE simulations.

In Fig.1, how RMSE performance of iterative MMSE improves with each iteration is shown. Overall, the RMSE decreases with more iterations. However, because MMSE is a biased estimator, the RMSE cannot keep decreasing all the way.

RMSE performance of iterative MMSE, MDS and MLE is compared and shown in Fig. 2 and Fig. 3, where the CRB is also shown. Detailed derivation for CRB expression for position estimators can be found in.[3]

In Fig. 2, $N = 25$ nodes are uniformly positioned within the given square area. And the performance is visualized by the CRB ellipse and the estimation variance ellipse. It is obvious that the iterative MMSE has the smallest variance ellipse, which is even smaller than the CRB ellipse. This means that the iterative MMSE has the best RMSE performance and can even break the CRB. For general conclusion, RMSE performance is observed for networks of different sizes in Fig. 3. And the above mentioned results still hold.

For study on the case of incomplete measurements, Fig. 4 shows performance ratio versus available paths ratio curve for a network of $N = 36$ unknown nodes. To clarify the results, the reference line of slope being

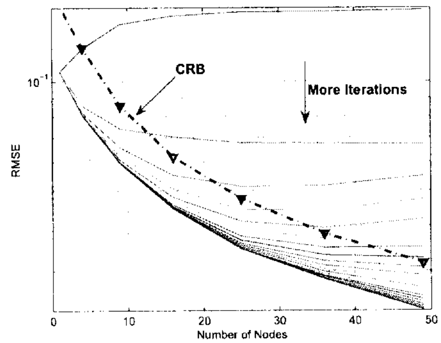
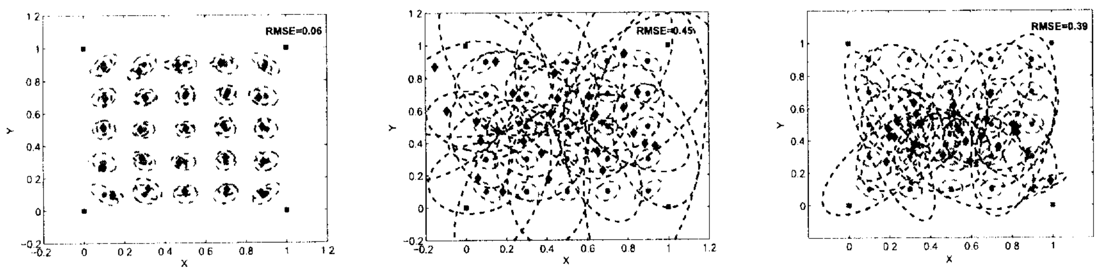


Figure 1. Performance of the iterative MMSE improves at each iteration.



(a) Iterative MMSE

(b) MDS

(c) MLE with random initial estimation

Figure 2. Performance comparison for $N = 25$ and $\frac{\sigma_{dB}^2}{n_p} = 1.7$. (*Blue squares*: anchor nodes. *Red dots*: true locations of unknown nodes. *Black diamonds*: the mean estimated locations. *Red dash ellipses*: CRB. *Black ellipses*: estimation variance.)

1 is also shown. The curve with incomplete measurements falls above the reference line. This means that, for example, when 80% paths are available, while 80% computation is needed, the performance only degrades by 10%. In other words, the performance degrades less than reduction of computation cost. Therefore, we can intentionally throw away certain paths and gain better computation efficiency at smaller performance sacrifice.

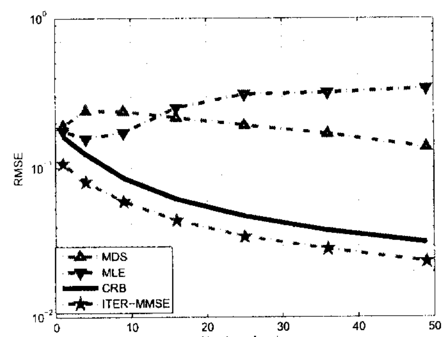


Figure 3. RMSE performance for different sizes of networks. ($\frac{\sigma_{dB}^2}{n_p} = 1.0$)

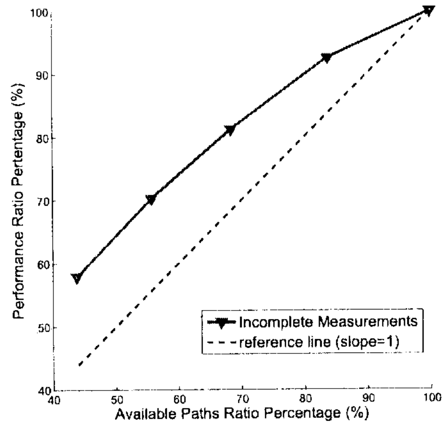


Figure 4. Performance ratio is defined as $\frac{\mathrm{RMSE}(L_t)}{\mathrm{RMSE}(L_a)}$. Available paths ratio is defined as $\frac{L_a}{L_t}$. Total number of nodes $N = 36$ and $\frac{\sigma^2_{dB}}{n_p} = 1.0$.

6. CONCLUSIONS

An iterative MMSE cooperative localization algorithm is proposed and studied in this work. To handle the complexity issue brought by the complicated multiple integrals and thus expand MMSE's application to large size network, we propose an iterative solution to implement MMSE cooperative estimator. The virtual priori PDF is adaptively adjusted with different iterations. We further examine the performance of the proposed algorithm in the situation where pair-wise range measurements are incomplete and proposed to reduce the computation burden by intentionally throwing away certain measurements. Numerical results show that the performance of the iterative MMSE beats existing classical cooperative localization techniques and even break though the CRB.

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