# ELC 4351: Digital Signal Processing

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# Outline

# Introduction

- Digital hardware: Digital computer and digital signal processor (DSP)

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- Software: Programmable operations

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- Digital hardware: Digital computer and digital signal processor (DSP)
- Software: Programmable operations
- A higher order of precision and robustness against noise, interference, uncertainty, etc.
- Sampling and quantization bring a distortion

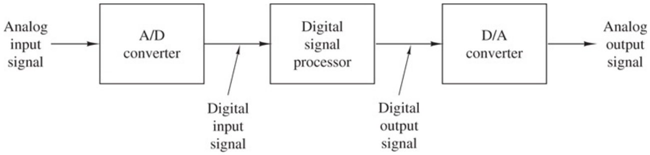


Figure 1.1.3 Block diagram of a digital signal processing system.

A signal is any physical quantity that varies with time, space, or any other independent variable or variables.

$$s_1(t) = 5t$$

$$s_2(t) = A\cos(2\pi f_c t + \theta)$$

$$s_3(x, y) = 2x + 4xy + 9y$$

$$s_1(nT_s) = 5nT_s, \quad t = nT_s, n = 0, 1, 2, \ldots$$

$$s[n] = 5nT_s$$

A system can perform an operation on a signal. Such operation is referred to as signal processing.

$$x(n) \longrightarrow^{F} y(n)$$

$$y(n) = F(x(n))$$

The system is characterized by the type of operation that it performs on the signal. For example, if the operation is linear, the system is called linear.

$$y(n) = \frac{1}{3}[x(n) + x(n-1) + x(n-2)]$$

# Classification of Signals

1. Multichannel and multidimensional signals

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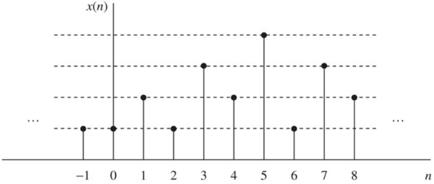


Figure 1.2.5 Digital signal with four different amplitude values.

# Classification of Signals

1. Multichannel and multidimensional signals
2. Continuous-time vs. discrete-time signals
3. Continuous-valued vs. discrete-valued signals

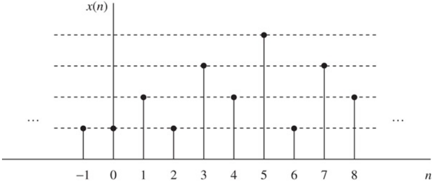


Figure 1.2.5 Digital signal with four different amplitude values.

4. Deterministic vs. random signals

# The Concept of Frequency



- The concept of frequency is directly related to the concept of time. It has the dimension of inverse time.

## Continuous-Time Sinusoidal Signals

$$x_a(t) = A\cos(\Omega t + \theta), \quad -\infty < t < \infty$$

$A$ is the amplitude of the sinusoid, $\Omega$ is the frequency in radians per second (rad/s), and $\theta$ is the phase in radians.

$$\Omega = 2\pi F$$

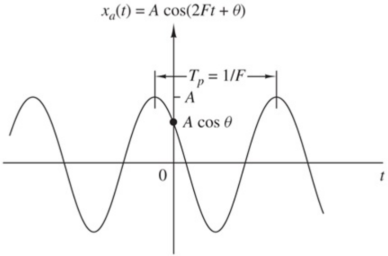


**Figure 1.3.1** Example of an analog sinusoidal signal.

# Continuous-Time Sinusoidal Signals



$x_a(t)$ is periodic with fundamental period $T_p = 1/F$.

$$x_a(t + T_p) = x_a(t)$$

Complex Exponential Signals

$$x_a(t) = Ae^{j(\Omega t+\theta)} = A\cos(\Omega t + \theta) + jA\sin(\Omega t + \theta)$$

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Q: Why use complex signal representation?

A: Easy to calculate $\frac{d}{dt}x_a(t)$ and $\int x_a(t)dt$.

Discrete-Time Sinusoidal Signals

$$x(n) = A\cos(\omega n + \theta), \quad -\infty < n < \infty$$

$n$ is the sample number, $A$ is the amplitude of the sinusoid, $\omega$ is the frequency in radians per sample, and $\theta$ is the phase in radians.

$$\omega = 2\pi f$$

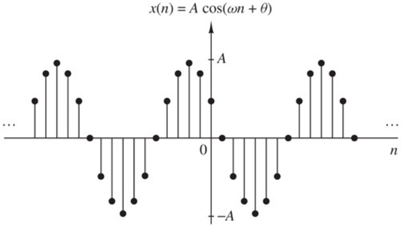



Figure 1.3.3 Example of a discrete-time sinusoidal signal ($\omega = \pi/6$ and $\theta = \pi/3$).

# Discrete-Time Sinusoidal Signals



- A discrete-time sinusoid is periodic only if its frequency $f$ is a rational number.

$$\cos(2\pi f(N+n)+\theta)=\cos(2\pi fn+\theta)$$

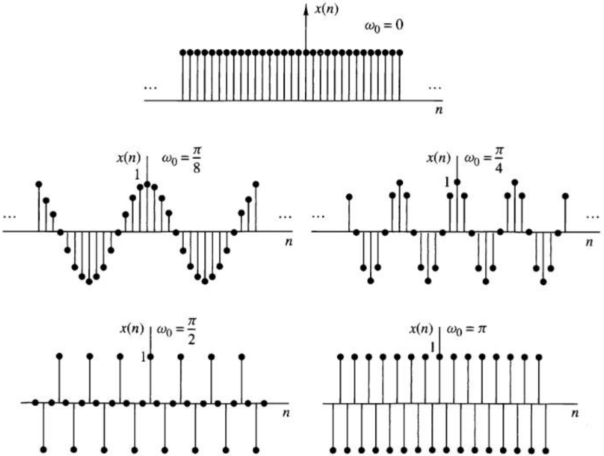
$$\Rightarrow 2\pi fN = 2k\pi \quad \Rightarrow \quad f=\frac{k}{N}$$

- Discrete-time sinusoids whose frequencies are separated by an integer multiple of $2\pi$ are identical.

$$\cos(\omega n+\theta)=\cos((\omega+2\pi)n+\theta)$$

# Discrete-Time Sinusoidal Signals

- The highest rate of oscillation in a discrete-time sinusoid is attained when $\omega = \pi$ (or $\omega = -\pi$).



# Discrete-Time Sinusoidal Signals



- The frequencies in any interval $\omega_1 \leq \omega \leq \omega_1 + 2\pi$ constitute all the existing discrete-time sinusoids or complex exponentials.
- The frequency range for discrete-time sinusoids is finite with duration $2\pi$.
- We choose the range $0 \leq \omega \leq 2\pi$ or $-\pi \leq \omega \leq \pi$ as the fundamental range.

# Harmonically Related Complex Exponentials

- Continuous-time Exponentials

The basic signals:

$$s_k(t) = e^{jk\Omega_0 t} = e^{j2\pi k F_0 t}, \quad k = 0, \pm 1, \pm 2, \ldots$$

$T_p = 1/F_0$ is a common period.

A linear combination of harmonically related complex exponentials

$$x_a(t) = \sum_{k=-\infty}^{\infty} c_k s_k(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\Omega_0 t}$$

where $c_k, k = 0, \pm 1, \pm 2, \ldots$ are arbitrary complex constants.

# Harmonically Related Complex Exponentials

$$\boxed{x_a(t) = \sum_{k=-\infty}^{\infty} c_k s_k(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\Omega_0 t}}$$

- Fourier series expansion for $x_a(t)$.
- The signal $x_a(t)$ is periodic with fundamental period $T_p = 1/F_0$.
- $\{c_k\}$ are the Fourier series coefficients.
- $s_k$ is the $k$th harmonic of $x_a(t)$.

# Harmonically Related Complex Exponentials

- Discrete-time Exponentials

  The basic signals:

$$s_k(n) = e^{j2\pi k f_0 n}, \quad k = 0, \pm 1, \pm 2, \ldots$$

  We choose $f_0 = 1/N$.

$$s_k(n) = e^{j2\pi kn/N}, \quad k = 0, 1, 2, \ldots, N-1$$

$$s_{k+N}(n) = e^{j2\pi n(k+N)/N} = e^{j2\pi n} s_k(n) = s_k(n)$$

# Harmonically Related Complex Exponentials



A linear combination of harmonically related complex exponentials

$$\boxed{x(n) = \sum_{k=0}^{N-1} c_k s_k(n) = \sum_{k=0}^{N-1} c_k e^{j2\pi kn/N}}$$

where $c_k, k = 0, 1, 2, \ldots, N-1$ are arbitrary complex constants.

- Fourier series expansion for discrete-time sequence $x(n)$.
- The signal $x(n)$ is periodic with fundamental period $N$.
- $\{c_k\}$ are the Fourier series coefficients.
- $s_k$ is the $k$th harmonic of $x(n)$.

# Analog-to-Digital (A/D) Converter

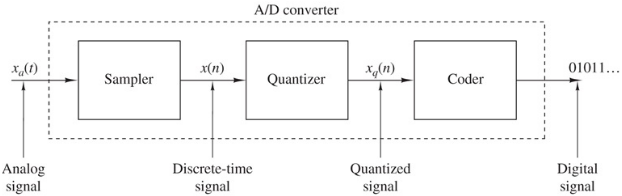



Figure 1.4.1 Basic parts of an analog-to-digital (A/D) converter.

1. Sampling: Conversion of a continuous-time signal into a discrete-time signal

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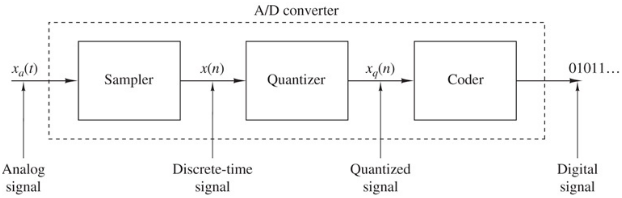


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2. Quantization: Conversion of a continuous-valued signal into a discrete-valued signal

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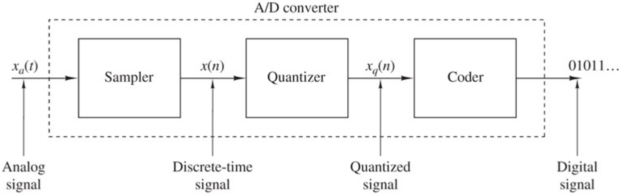


**Figure 1.4.1** Basic parts of an analog-to-digital (A/D) converter.

1. Sampling: Conversion of a continuous-time signal into a discrete-time signal
2. Quantization: Conversion of a continuous-valued signal into a discrete-valued signal
3. Coding: Each discrete-valued sample is represented by a $b$-bit binary sequence

# Quiz – August 25, 2016



- If $x_a(t)$ is a continuous-time periodic signal with fundamental period $T_p = 1/F_0$, write $x_a(t)$ in terms of its Fourier series expansion.

- What are the three major processes of an analog-to-digital conversion?