ELC 4351: Digital Signal Processing

Windowing and Window Design

Prof. Liang Dong

Baylor University
SCHOOL OF ENGINEERING & COMPUTER SCIENCE
Department of Electrical & Computer Engineering

Windowing

- Windowing: Truncate the impulse response $h(n)$ of ideal frequency-selective filters.
- For example, an ideal (desired) LPF $H(\omega)$:

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega}, & |\omega| \le \omega_c \\ 0, & \omega_c < |\omega| \le \pi \end{cases}$$

where α is the sample delay. This is a linear-phase filter.

$$x(n) \longrightarrow \boxed{h(n)} \longrightarrow x(n-\alpha)$$

Windowing

- Therefore, the time-domain ideal (desired) impulse response $h_d(n)$ is

$$\begin{aligned} h_d(n) &= \mathcal{F}^{-1}\left[H_d(e^{j\omega})\right] \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega \\ &= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\alpha\omega}e^{j\omega n}d\omega \\ &= \frac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)} \end{aligned}$$

- $h_d(n)$ is a sinc function symmetric with respect to α.

Windowing

- We want to truncate $h_d(n)$ on both sides to obtain a causal and linear-phase FIR filter $h(n)$ of length M. Therefore,

$$h(n) = \begin{cases} h_d(n), & 0 \le n \le M-1 \\ 0, & \text{elsewhere} \end{cases} \quad \text{and} \quad \alpha = \frac{M-1}{2}$$

- Windowing: $h(n)$ can be thought of as being formed by the product of $h_d(n)$ and a window function $w(n)$ as

$$h(n) = h_d(n)w(n)$$

where

$$w(n) = \begin{cases} 1, & 0 \le n \le M-1 \\ 0, & \text{elsewhere} \end{cases}$$

Windowing

- Rectangular window function

$$w(n) = \begin{cases} 1, & 0 \le n \le M-1 \\ 0, & \text{elsewhere} \end{cases}$$

- In the frequency domain, the causal FIR filter response $H(e^{j\omega})$ is

$$\begin{aligned} H(e^{j\omega}) &= H_d(e^{j\omega}) \otimes W(e^{j\omega}) \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi} W(e^{j\lambda}) H_d(e^{j(\omega-\lambda)}) d\lambda \end{aligned}$$

Windowing Operation in the Frequency Domain

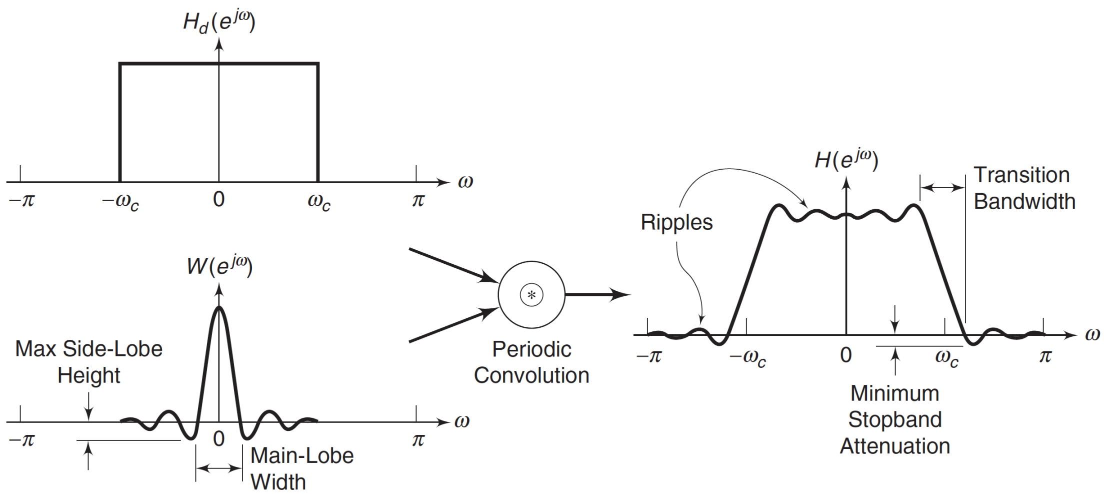

Figure: Window $w(n)$ has a finite length of M. Its response $W(\omega)$ has main-lobe width proportional to $1/M$. The transition width of $H_d(\omega)$ is proportional to $1/M$. The side lobes produce ripples in passband and stopband of $H_d(\omega)$.

Window Functions

- A general window function

$$w(n) = \begin{cases} \text{smooth symmetric function w.r.t. } \alpha, & 0 \leq n \leq M-1 \\ 0, & \text{elsewhere} \end{cases}$$

- A list of window functions