ELC 4350: Principles of Communication

Spatial Diversity and Spatial Multiplexing

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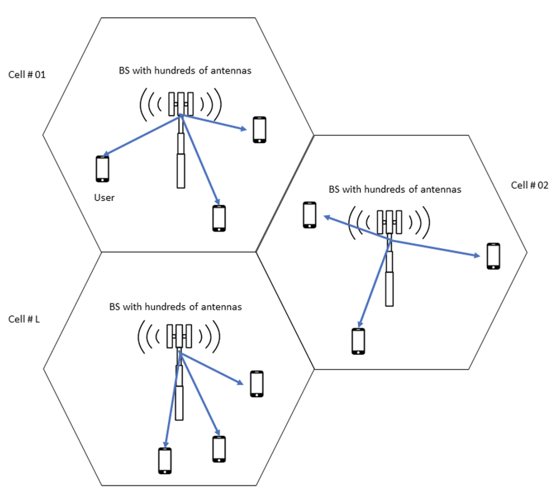
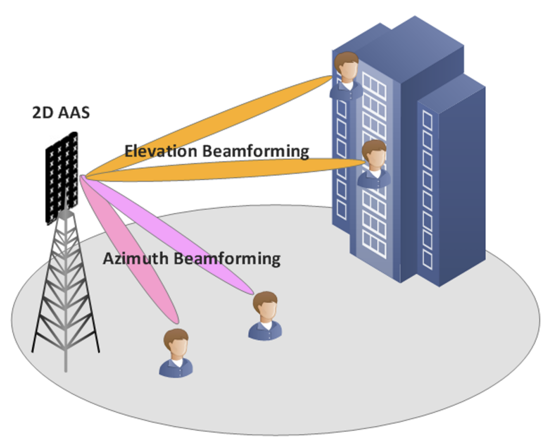
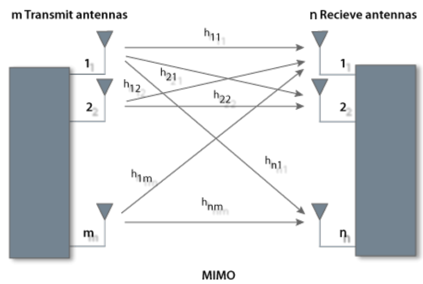
Multiple Antennas for Communication Systems





Multiple-Input Multiple-Output (MIMO) Systems



- Multiple antennas at the transmitter and at the receiver – Multi-Input Multi-Output (MIMO) Communication System
- Spatial Diversity
- Spatial Multiplexing

Spatial Diversity – Beamforming

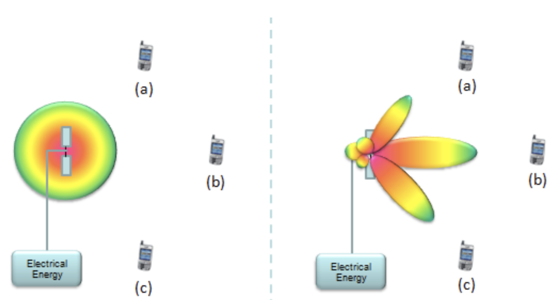


Figure: Beamforming. We consider a downlink to multiple mobile users. Each user has a single antenna.

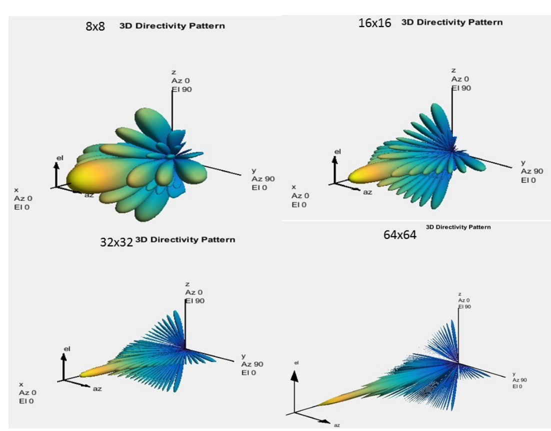


Figure: 3D Directivity Pattern.

Downlink Beamforming

- Transmitted signal at the BS

$$\mathbf{x} = \mathbf{w}_1 s_1 + \mathbf{w}_2 s_2$$

where $\mathbf{w}_1$ and $\mathbf{w}_2$ are the beamforming weights for user 1 signal s_1 and user 2 signal s_2, respectively. Usually, $\|\mathbf{w}_i\| = 1$.

- User 1 received signal

$$\begin{aligned} y_1 &= \mathbf{h}_1^H \mathbf{x} + n_1 \\ &= \mathbf{h}_1^H (\mathbf{w}_1 s_1 + \mathbf{w}_2 s_2) + n_1 \\ &= \underbrace{\mathbf{h}_1^H \mathbf{w}_1 s_1}_{\text{signal}} + \underbrace{\mathbf{h}_1^H \mathbf{w}_2 s_2}_{\text{interference}} + n_1 \end{aligned}$$

- Signal-to-interference-plus-noise ratio (SINR) at User 1

$$\text{SINR}_1 = \frac{|\mathbf{h}_1^H \mathbf{w}_1|^2}{|\mathbf{h}_1^H \mathbf{w}_2|^2 + \sigma_n^2}$$

Downlink Beamforming

- Similarly, User 2 received signal

$$y_2 = \underbrace{\mathbf{h}_2^H \mathbf{w}_2 s_2}_{\text{signal}} + \underbrace{\mathbf{h}_2^H \mathbf{w}_1 s_1}_{\text{interference}} + n_2$$

- Signal-to-interference-plus-noise ratio (SINR) at User 2

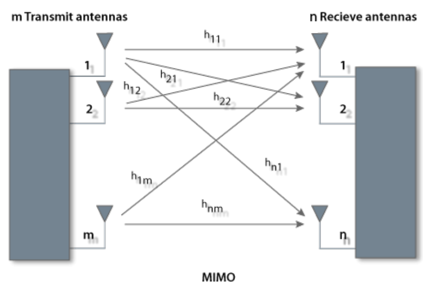
$$\text{SINR}_2 = \frac{|\mathbf{h}_2^H \mathbf{w}_2|^2}{|\mathbf{h}_2^H \mathbf{w}_1|^2 + \sigma_n^2}$$

- Beamforming weights design:

$$\mathbf{w}_1 = \frac{\mathbf{h}_1}{\|\mathbf{h}_1\|}, \quad \mathbf{w}_2 = \frac{\mathbf{h}_2}{\|\mathbf{h}_2\|}$$

- Zero-forcing beamforming: $\mathbf{w}_1 \perp \mathbf{h}_2$, $\mathbf{w}_2 \perp \mathbf{h}_1$.

MIMO Multiplexing



- Transmitted signal at the BS is

$$\mathbf{x} = \mathbf{W}\mathbf{s}, \quad \text{where } \mathbf{s} = [s_1, s_2, \ldots, s_{M_t}]^T$$

- The receiver has multiple antennas. The received signal is

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} = \mathbf{H}\mathbf{W}\mathbf{s} + \mathbf{n}$$

MIMO Multiplexing

- Perform singular value decomposition (SVD) on an $(M_r \times M_t)$ channel matrix $\mathbf{H}$

$$\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H$$

- $\mathbf{U}$ is an $M_r \times M_r$ complex unitary matrix, $\boldsymbol{\Sigma}$ is an $M_r \times M_t$ rectangular diagonal matrix with non-negative real numbers on the diagonal, and $\mathbf{V}$ is an $M_t \times M_t$ complex unitary matrix.

$$\mathbf{U}^H\mathbf{U} = \mathbf{U}\mathbf{U}^H = \mathbf{I}_{M_r}, \quad \mathbf{V}^H\mathbf{V} = \mathbf{V}\mathbf{V}^H = \mathbf{I}_{M_t}$$

- The SVD can be written as

$$\mathbf{H} = \sum_{i=1}^{r} \sigma_i \mathbf{u}_i \mathbf{v}_i^H$$

  where σ_i is the ith diagonal element of $\boldsymbol{\Sigma}$, $\mathbf{u}_i$ is the ith column of $\mathbf{U}$, and $\mathbf{v}_i$ is the ith column of $\mathbf{V}$.
  $r \leq \min\{M_r, M_t\}$ is the rank of $\mathbf{H}$.

MIMO Multiplexing

- (Spatial) filtering on the received signal

$$\begin{aligned}\mathbf{U}^H\mathbf{y} &= \mathbf{U}^H\mathbf{H}\mathbf{x} + \mathbf{U}^H\mathbf{n} \\ &= \mathbf{U}^H\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H\mathbf{x} + \mathbf{U}^H\mathbf{n} \\ &= \boldsymbol{\Sigma}\mathbf{V}^H\mathbf{x} + \mathbf{U}^H\mathbf{n}\end{aligned}$$

- Let $\mathbf{x} = \mathbf{W}\mathbf{s} = \mathbf{V}\mathbf{s}$, $\mathbf{y}' = \mathbf{U}^H\mathbf{y}$, and $\mathbf{n}' = \mathbf{U}^H\mathbf{n}$, (rotation of vectors),
  we have

$$\mathbf{y}' = \boldsymbol{\Sigma}\mathbf{s} + \mathbf{n}'$$

- Example:

$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} + \begin{bmatrix} n_1' \\ n_2' \end{bmatrix}$$

$$\begin{aligned} y_1' &= \sigma_1 s_1 + n_1' \\ y_2' &= \sigma_2 s_2 + n_2' \end{aligned}$$