ELC 4350: Principles of Communication

Baseband Data Transmission

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Baseband Communication Blockdiagram

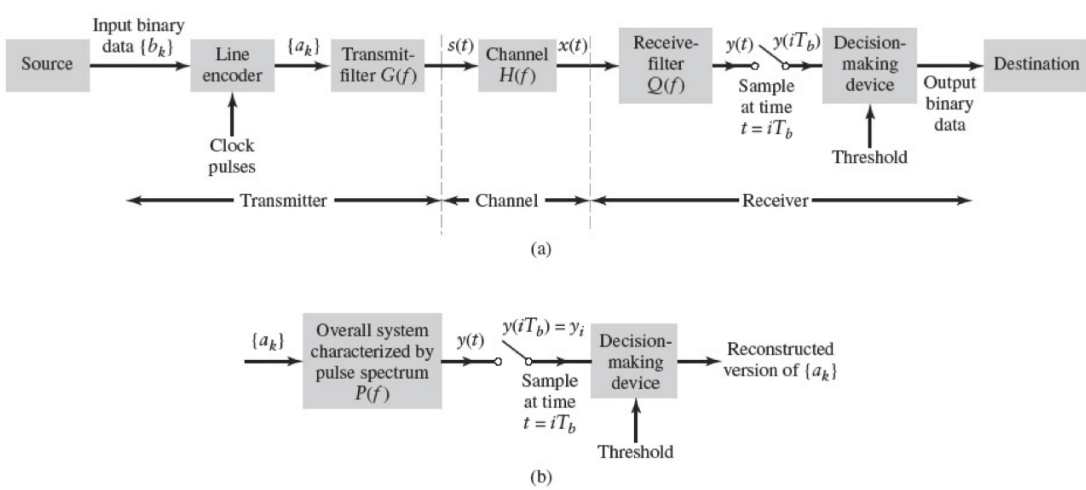
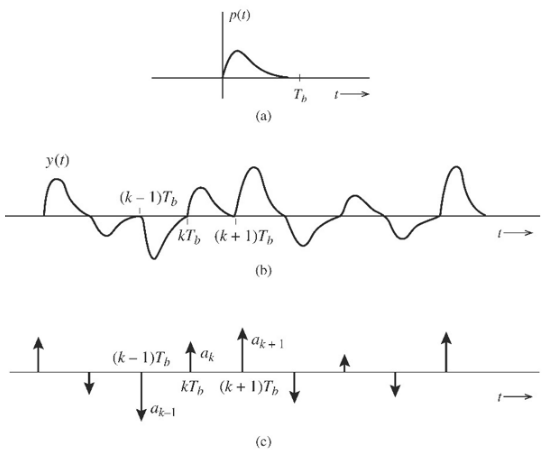


Figure: (a) Block Diagram of Baseband Communication Systems. (b) Simplified Block Diagram.

Baseband Modulation / Baseband Line Coding



- The baseband signal

$$y(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT_b)$$

where $\{a_k\}$ are symbols, T_b is bit duration (or symbol duration), and $p(t)$ is the overall pulse.

Baseband Modulation / Baseband Line Coding

- The baseband transmitted signal

$$s(t) = \sum_{k=-\infty}^{\infty} a_k g(t - kT_b)$$

where $g(t)$ is the pulse shaping filter at the transmitter.

- The baseband received signal

$$x(t) = s(t) \otimes h(t)$$

where $h(t)$ is the channel impulse response.

- The output of receive-filter

$$y(t) = x(t) \otimes q(t)$$

where $q(t)$ is the impulse response of receive-filter.

Baseband Pulse Shaping

- Overall pulse shape - Transmitter filter, Linear Communication Channel, Receiver filter

$$p(t) = g(t) \otimes h(t) \otimes q(t)$$

- Fourier transform of the pulse

$$P(f) = G(f)H(f)Q(f)$$

The transmit-pulse $G(f)$ and the receive-filter $Q(f)$ can conserve the communication bandwidth.

Intersymbol Interference (ISI)

- The receive-filter output $y(t)$ is sampled synchronously with the transmitter

$$y(iT_b) = \sum_{k=-\infty}^{\infty} a_k p[(i-k)T_b]$$

- Discrete convolution sum: $y_i = \sum_{k=-\infty}^{\infty} a_k p_{i-k}$
- Assume that $p(0) = \sqrt{E}$, where E is the transmitted signal energy per symbol.
- Therefore

$$y_i = \sqrt{E} a_i + \underbrace{\sum_{k=-\infty, k \neq i}^{\infty} a_k p_{i-k}}_{\text{intersymbol interference (ISI)}}$$

Nyquist Criterion for Zero ISI

- The Nyquist's criterion for zero ISI

$$p_i = p(iT_b) = \begin{cases} \sqrt{E} & , i = 0 \\ 0 & , i \neq 0 \end{cases}$$

- The optimum pulse shape is the sinc function

$$p_{\text{opt}}(t) = \sqrt{E}\,\mathrm{sinc}(2B_0 t) = \frac{\sqrt{E}\sin(2\pi B_0 t)}{2\pi B_0 t}$$

where $B_0 = \frac{1}{2T_b} = \frac{R_b}{2}$. (Nyquist bandwidth – minimum transmission bandwidth for zero ISI.)

- The Fourier transform of the optimum pulse is

$$P_{\text{opt}}(f) = \begin{cases} \frac{\sqrt{E}}{2B_0} & , -B_0 < f < B_0 \\ 0 & , \text{otherwise} \end{cases}$$

Optimum Pulse Shaping Filter

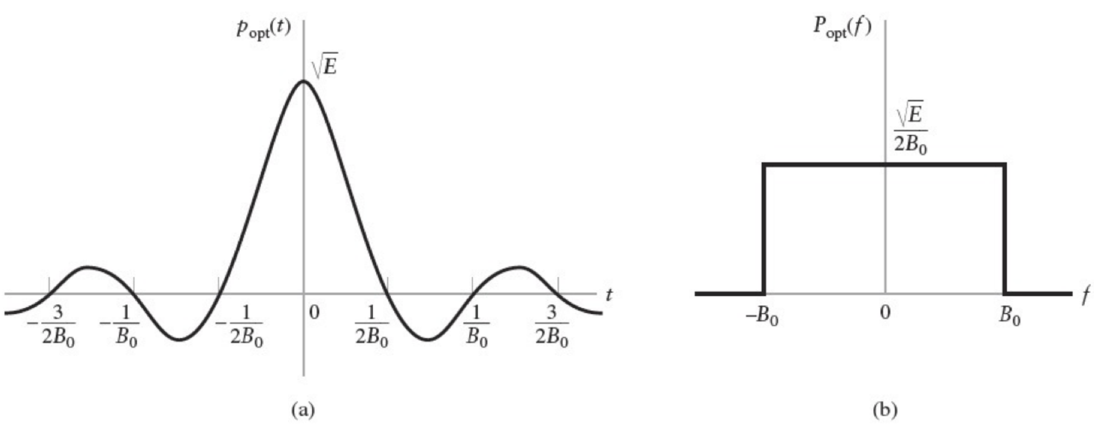
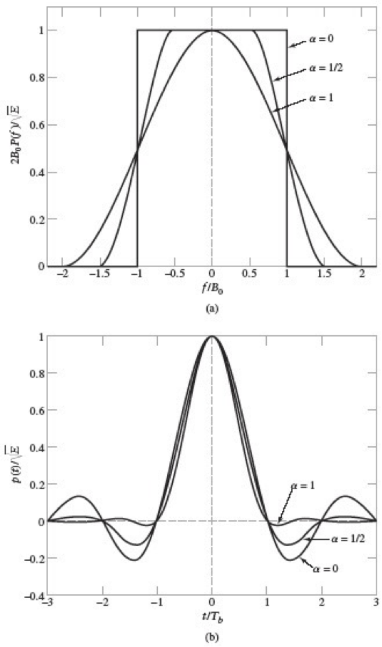


Figure: (a) Optimum pulse shape sinc function $p(t)$. (b) Optimum filter "brick-wall" $P(f)$.

Time pulse function $p(t)$ decreases as $1/|t|$. — Slow rate of decay.

Raised-Cosine Pulse Spectrum



- Flat portion, $0 \le |f| \le f_1$
- Roll-off portion, $f_1 < |f| < 2B_0 - f_1$

(a) Raised-cosine pulse spectrum $P(f)$. (b) Raise-cosine pulse $p(t)$.

Raised-Cosine Pulse

- Raised-cosine pulse spectrum

$$P(f) = \begin{cases} \frac{\sqrt{E}}{2B_0} & , 0 \le |f| < f_1 \\ \frac{\sqrt{E}}{4B_0}\left\{1 + \cos\left[\frac{\pi(|f| - f_1)}{2(B_0 - f_1)}\right]\right\} & , f_1 \le |f| < 2B_0 - f_1 \\ 0 & , 2B_0 - f_1 \le |f| \end{cases}$$

where the roll-off factor $\alpha = 1 - f_1/B_0$.

- Raised-cosine pulse (inverse Fourier transform of the raised-cosine pulse spectrum)

$$p(t) = \sqrt{E}\,\mathrm{sinc}(2B_0 t)\left(\frac{\cos(2\pi\alpha B_0 t)}{1 - 16\alpha^2 B_0^2 t^2}\right)$$

Transmission Bandwidth Requirement

- The transmission bandwidth required by using the raised-cosine pulse spectrum is

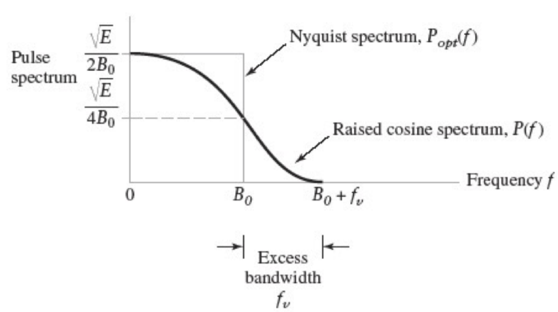
$$B_T = 2B_0 - f_1 = B_0(1 + \alpha)$$

- Excess bandwidth

$$B_T = B_0 + \underbrace{\alpha B_0}_{\text{excess bandwdith}}$$

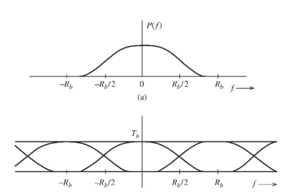
Raised-Cosine Pulse Spectrum Properties

- The roll-off portion of the spectrum $P(f)$ is odd symmetry about the midpoints $f = \pm B_0$.



- The infinite sum of replicas of the raised-cosine pulse spectrum spaced by $2B_0$ is a constant

$$\sum_{m=-\infty}^{\infty} P(f - 2mB_0) = \frac{\sqrt{E}}{2B_0}$$



Root Raised-Cosine Pulse Spectrum

- The combination of transmit-filter and channel is a root raised-cosine

$$G(f)H(f) = \sqrt{P(f)}$$

- The receive-filter is a root raised-cosine

$$Q(f) = \sqrt{P(f)}$$

- Therefore

$$G(f)H(f)Q(f) = P(f)$$

Baseband Transmission of M-ary PAM

- Baud rate is symbol rate. 1 baud is equal to $\log_2 M$ bits per second.
- T_s is symbol duration, and T_b is bit duration.

$$T_s = T_b \log_2 M$$

- To maintain the same received SNR, the transmitted power of a M-ary PAM system must be increased by a factor of $M^2/\log_2 M$, compared to a binary PAM system.

The Eye Diagram

- Synchronized superposition of many successive symbol intervals of the distorted waveform appearing at the output of the receive-filter.

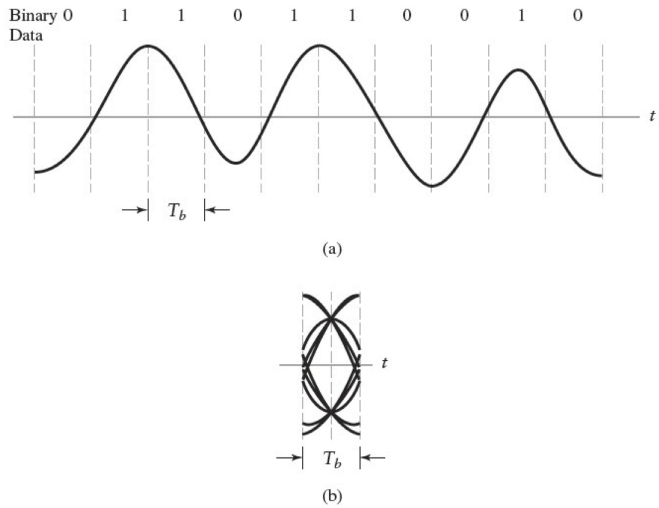


Figure: (a) Binary data sequence waveform. (b) Eye pattern formed by superposition.

Reading an Eye Diagram

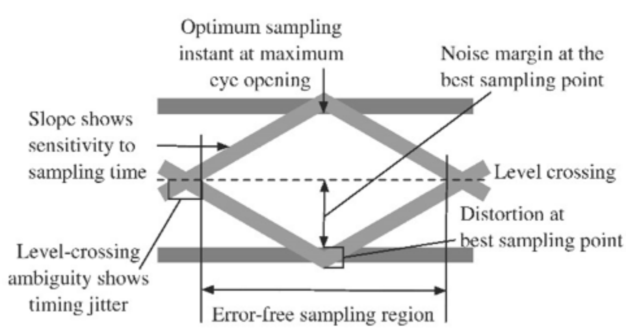


Figure: Interpretation of the eye diagram for a baseband binary PAM system.

Eye Diagram Example

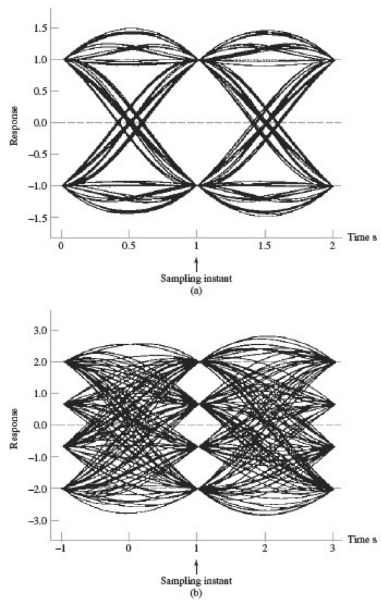


Figure: Eye diagram of received signal with no noise. (a) $M = 2$. (b) $M = 4$.

Eye Diagram Example

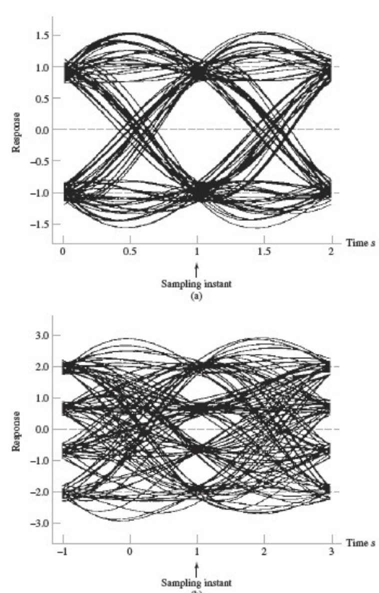


Figure: Eye diagram of received signal with noise. (a) $M = 2$. (b) $M = 4$.

Timing Recovery

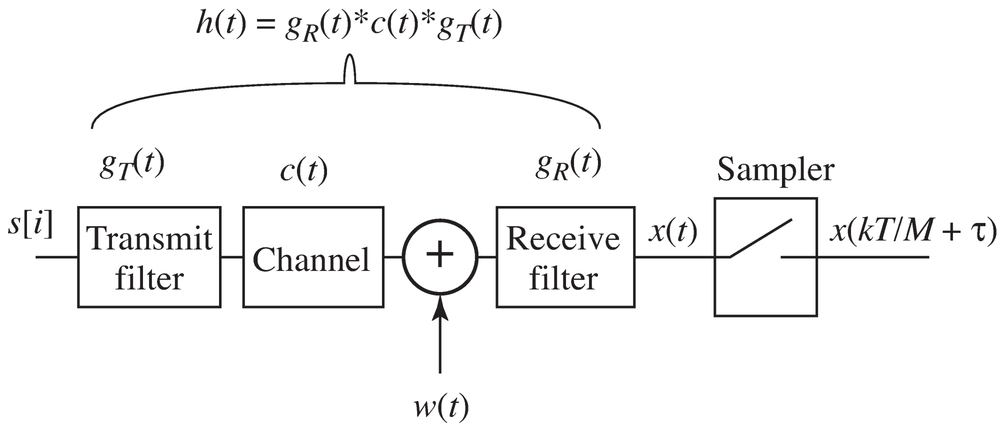


Figure: Transfer function h combines the effects of the transmitter pulse shaping, the channel, and the receiver filter. Receiver samples at $kT/M + \tau$.

Timing Recovery

- The sampled output

$$x\left(\frac{kT}{M}+\tau\right) = \sum_{i=-\infty}^{\infty} s[i]h(t-iT) + w(t) \otimes g_R(t)\Big|_{t=kT/M+\tau}$$

$$x[k] = x\left(\frac{kT}{M}+\tau\right) = \sum_{i=-\infty}^{\infty} s[i]h\left(\frac{kT}{M}+\tau-iT\right) + v(t)$$

Decision-Directed Timing Recovery

- Timing recovery by minimizing the cluster variance

$$J_{CV}(\tau) = \text{avg}\{(Q(x[k]) - x[k])^2\}$$

where $Q()$ is to map to the nearest symbol value (quantization).

- Iteratively solving for τ that minimizes $J_{CV}(\tau)$.
Update equation:

$$\tau[k+1] = \tau[k] - \mu' \left. \frac{dJ_{CV}(\tau)}{d\tau} \right|_{\tau=\tau[k]}$$

This is the Gradient Decent method, where μ' is the step size.

Decision-Directed Timing Recovery

- The approximation of the derivative is (approximation because we swap the order of the derivative and the average)

$$\begin{aligned} \frac{dJ_{CV}(\tau)}{d\tau} &\approx \text{avg}\left\{\frac{\mathrm{d}(\mathrm{Q}(\mathrm{x}[\mathrm{k}]) - \mathrm{x}[\mathrm{k}])^2}{\mathrm{d}\tau}\right\} \\ &= -2\text{avg}\left\{(Q(x[k]) - x[k])\frac{dx[k]}{d\tau}\right\} \end{aligned}$$

- Numerically approximating $dx[k]/d\tau$ as

$$\frac{dx[k]}{d\tau} = \frac{dx(kT/M + \tau)}{d\tau} \approx \frac{x(kT/M + \tau + \delta) - x(kT/M + \tau - \delta)}{2\delta}$$

which is valid for small δ.

Decision-Directed Timing Recovery

- The update equation becomes

$$\tau[k+1] = \tau[k] + \mu \cdot \text{avg}\left\{(Q(x[k]) - x[k]) \cdot \left[x\left(\frac{kT}{M} + \tau[k] + \delta\right) - x\left(\frac{kT}{M} + \tau[k] - \delta\right)\right]\right\}$$

  where $\mu = \mu'/\delta$.

- $x(kT/M + \tau[k] + \delta)$ and $x(kT/M + \tau[k] - \delta)$ can be interpolated from the neighborhood of $x(kT/M + \tau[k])$.

- If the $\tau[k]$ values are too noisy, the step size μ can be decreased.

Decision-Directed Timing Recovery

- Using Stochastic Gradient Decent method, we simplify the update equation as

$$\tau[k+1] = \tau[k] + \mu \cdot \cancel{\text{avg}}\left\{(Q(x[k]) - x[k]) \cdot \left[x\left(\frac{kT}{M} + \tau[k] + \delta\right) - x\left(\frac{kT}{M} + \tau[k] - \delta\right)\right]\right\}$$

Decision-Directed Timing Recovery

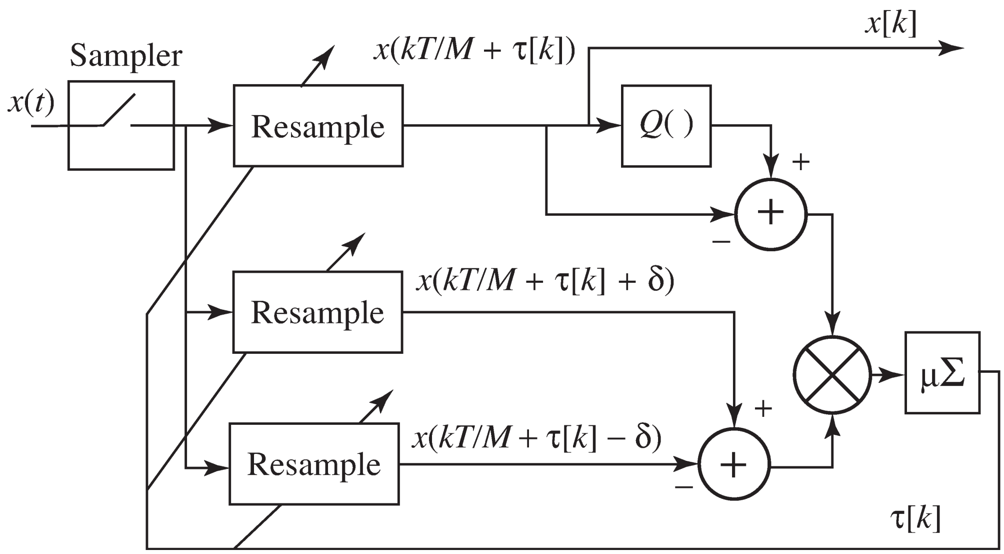


Figure: Timing recovery that minimizes the cluster variance. Digital interpolations and resamplers.

Matched Filter

- The transmit-filter $g_T(t)$ and the receive-filter $g_R(t)$ are matched filters.
- Correlating the received signal with exact the signal shape of the transmit-filter. This is equivalent to convolving the received signal with a conjugate time-reversed version of the transmit-filter.

$$\boxed{g_R(t) = g_T^*(-t)}$$

- The matched filter is the optimal linear filter for maximizing the signal-to-noise ratio (SNR) in the presence of additive random noise.

Matched Filter – Derivation

- The output y of a linear filter g with the input signal x is

$$y[n] = \sum_{k=-\infty}^{\infty} g[n-k]x[k], \text{ or } y(t) = \int_{\tau} g(t-\tau)x(\tau)d\tau$$

- Using signal vector representation, we check a particular output

$$y = y[0] = \sum_{k=-\infty}^{\infty} g[-k]x[k] = \sum_{k=-\infty}^{\infty} h^*[k]x[k] = \mathbf{h}^H\mathbf{x}$$

Matched Filter – Derivation

- Signal x includes the desirable signal s and additive random noise w

$$\mathbf{x} = \mathbf{s} + \mathbf{w}$$

- The filter output is

$$y = \mathbf{h}^H\mathbf{x} = \underbrace{\mathbf{h}^H\mathbf{s}}_{\text{signal component}} + \underbrace{\mathbf{h}^H\mathbf{w}}_{\text{noise component}}$$

- The SNR is

$$\text{SNR} = \frac{|\mathbf{h}^H\mathbf{s}|^2}{\mathrm{E}\{|\mathbf{h}^H\mathbf{w}|^2\}} = \frac{|\mathbf{h}^H\mathbf{s}|^2}{\mathrm{E}\{(\mathbf{h}^H\mathbf{w})(\mathbf{h}^H\mathbf{w})^H\}} = \frac{|\mathbf{h}^H\mathbf{s}|^2}{\mathbf{h}^H\mathrm{E}\{\mathbf{w}\mathbf{w}^H\}\mathbf{h}}$$

Matched Filter – Derivation

- The covariance matrix of noise is Hermitian symmetry

$$R_{\mathbf{w}} = \mathrm{E}\{\mathbf{w}\mathbf{w}^H\}, \qquad R_{\mathbf{w}}^H = R_{\mathbf{w}}$$

- The SNR is

$$\begin{aligned}
\text{SNR} &= \frac{|\mathbf{h}^H\mathbf{s}|^2}{\mathbf{h}^H R_{\mathbf{w}}\mathbf{h}} \\
&= \frac{|(R_{\mathbf{w}}^{1/2}\mathbf{h})^H(R_{\mathbf{w}}^{-1/2}\mathbf{s})|^2}{(R_{\mathbf{w}}^{1/2}\mathbf{h})^H(R_{\mathbf{w}}^{1/2}\mathbf{h})} \\
&\le \frac{\left[(R_{\mathbf{w}}^{1/2}\mathbf{h})^H(R_{\mathbf{w}}^{1/2}\mathbf{h})\right]\left[(R_{\mathbf{w}}^{-1/2}\mathbf{s})^H(R_{\mathbf{w}}^{-1/2}\mathbf{s})\right]}{(R_{\mathbf{w}}^{1/2}\mathbf{h})^H(R_{\mathbf{w}}^{1/2}\mathbf{h})} \\
&= \mathbf{s}^H R_{\mathbf{w}}^{-1}\mathbf{s}
\end{aligned}$$

The inequality is the Cauchy-Schwarz inequality: $|\mathbf{a}^H\mathbf{b}|^2 \le (\mathbf{a}^H\mathbf{a})(\mathbf{b}^H\mathbf{b})$. It is equal only when $\mathbf{b} = \rho\mathbf{a}$, ρ real number.

Matched Filter – Derivation

- Therefore, the maximum SNR is achieved when

$$R_{\mathbf{w}}^{1/2}\mathbf{h} = \rho R_{\mathbf{w}}^{-1/2}\mathbf{s}$$

- We have the optimal linear filter as

$$\mathbf{h} = \rho R_{\mathbf{w}}^{-1}\mathbf{s}$$

- Finally, the (optimal) linear filter $g[k] = h^*[-k]$ is the complex-conjugate time-reversal of the desired signal s.

Linear Equalization

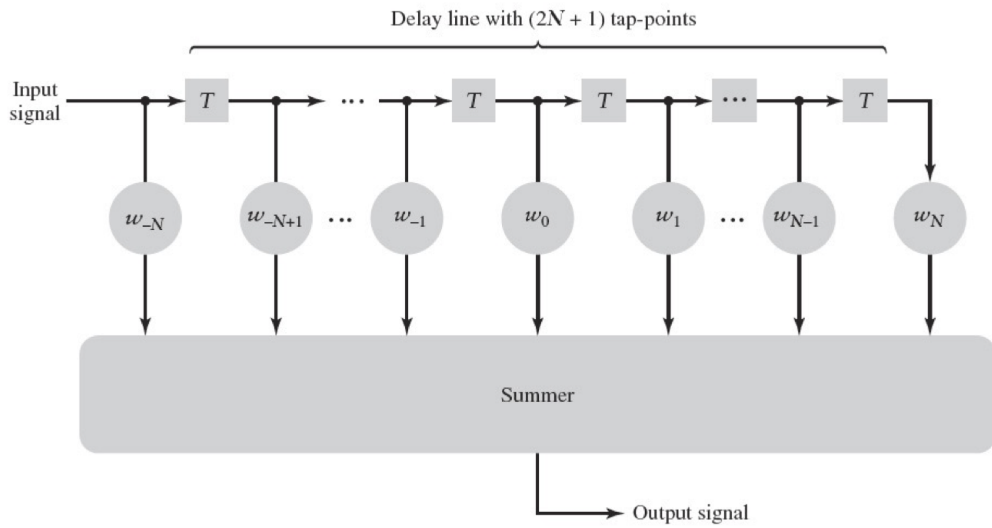


Figure: Adjustable Transversal Equalizer. (a) Delay line whose taps are uniformly spaced with symbol duration T. (b) (2N+1) Adjustable weights $\{w\}$ (with structural symmetry).

Zero-Forcing Equalization

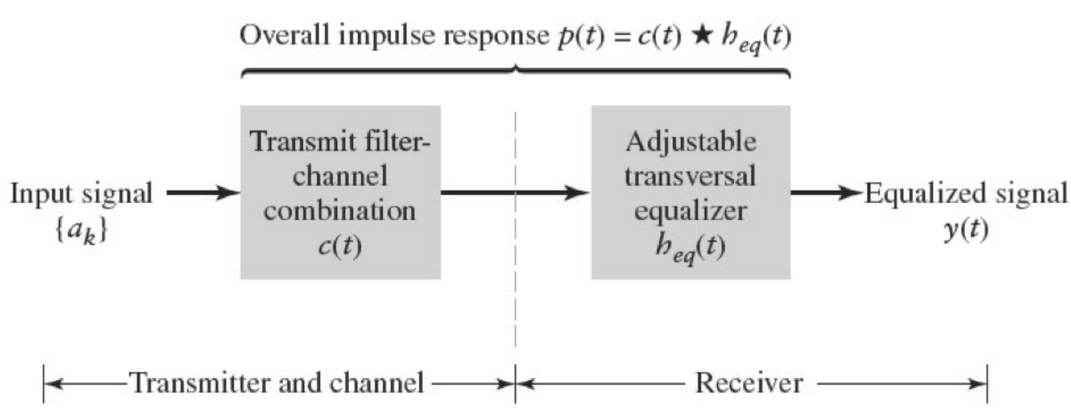


Figure: Channel Equalization. (a) First subsystem represents the combined action of the transmit-filter and the communication channel. (b) Second subsystem accounts for pulse shaping combined with distortion equalization in the receiver.

Zero-Forcing Equalization

- Impulse response of the equalizer

$$h_{\text{eq}}(t) = \sum_{k=-N}^{N} w_k \delta(t - kT)$$

- Overall impulse response of the cascade filters

$$\begin{aligned} p(t) &= c(t) \otimes h_{\text{eq}}(t) \\ &= c(t) \otimes \sum_{k=-N}^{N} w_k \delta(t - kT) \\ &= \sum_{k=-N}^{N} w_k c(t) \otimes \delta(t - kT) \\ &= \sum_{k=-N}^{N} w_k c(t - kT) \end{aligned}$$

Zero-Forcing Equalization

- Discrete convolution sum

$$\begin{aligned} p(iT) &= \sum_{k=-N}^{N} w_k c((i-k)T) \\ p_i &= \sum_{k=-N}^{N} w_k c_{i-k} \end{aligned}$$

- Nyquist criterion to eliminate ISI

$$p_i = \begin{cases} \sqrt{E} & , i = 0 \\ 0 & , i \neq 0 \longrightarrow i = \pm 1, \pm 2, \ldots, \pm N \end{cases}$$

Zero-Forcing Equalization

- We obtain a system of (2N+1) simultaneous equations:

$$\sum_{k=-N}^{N} w_k c_{i-k} = \begin{cases} \sqrt{E} & ,i=0 \\ 0 & ,i=\pm 1, \pm 2, \ldots, \pm N \end{cases}$$

- In matrix form:

$$\underbrace{\begin{bmatrix} c_0 & \cdots & c_{-N+1} & c_{-N} & \cdots & c_{-2N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{N-1} & \cdots & c_0 & c_{-1} & \cdots & c_{-N-1} \\ c_N & \cdots & c_1 & c_0 & \cdots & c_{-N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{2N} & \cdots & c_{N+1} & c_N & \cdots & c_0 \end{bmatrix}}_{\text{Toeplitz matrix } \mathbf{C}} \underbrace{\begin{bmatrix} w_{-N} \\ \vdots \\ w_{-1} \\ w_0 \\ \vdots \\ w_N \end{bmatrix}}_{\mathbf{w}} = \underbrace{\begin{bmatrix} 0 \\ \vdots \\ 0 \\ \sqrt{E} \\ \vdots \\ 0 \end{bmatrix}}_{\mathbf{b}}$$

Zero-Forcing Equalization

- We have

$$\mathbf{C} = \mathbf{w}\mathbf{b}$$

- Therefore, the weights of the zero-forcing equalizer (linear filter tapped delay line) are

$$\boxed{\mathbf{w} = \mathbf{C}^{-1}\mathbf{b}}$$

- The set of coefficient $\{c_k\}_{k=-N}^{N}$ can be obtained by sending pseudo-noise (PN) sequence as pilot signals to the receiver.

- The PN sequence is known *a priori* to the receiver.

Minimum Mean Square Error Equalization

- The baseband discrete-time received signal is

$$\begin{aligned} r(iT) &= \sum_{k=0}^{N} c_k s((i-k)T) + n(iT) \\ r_i &= \sum_{k=0}^{N} c_k s_{i-k} + n_i \end{aligned}$$

where $\{c_k\}$ are complex-valued channel taps, N is the channel length, $\{s_i\}$ are the complex-valued symbols, and n_i is the complex-valued AWGN with $\mathrm{E}[|n_i|^2] = \sigma_n^2$.

Minimum Mean Squared Error Equalization

- The linear equalization is given by

$$y_i = \sum_{k=0}^{M} w_k^* r_{i-k} = \mathbf{w}^H \mathbf{r}_i$$

where $\{w_k\}$ are complex-valued equalizer weights, M is the equalizer order, $\mathbf{w} = [w_0, w_1, \ldots, w_M]^T$ and $\mathbf{r}_i = [r_i, r_{i-1}, \ldots, r_{i-M}]^T$.

- The received signal vector $\mathbf{r}_i$ is

$$\mathbf{r}_i = \mathbf{C}\mathbf{s}_i + \mathbf{n}_i$$

where $\mathbf{s}_i = [s_i, s_{i-1}, \ldots, s_{i-L}]^T$, $\mathbf{n}_i = [n_i, n_{i-1}, \ldots, n_{i-M}]^T$, $L = N + M$, and the channel matrix $\mathbf{C} \cdots$

Minimum Mean Squared Error Equalization

- The received signal vector $\mathbf{r}_i$ is

$$\mathbf{r}_i = \mathbf{C}\mathbf{s}_i + \mathbf{n}_i$$

$\cdots$ and $\mathbf{C}$ is a dimension $(M+1) \times (L+1)$ Teoplitz matrix

$$\mathbf{C} = \begin{bmatrix} c_0 & c_1 & \cdots & c_M & 0 & \cdots & 0 \\ 0 & c_0 & \ddots & c_{M-1} & c_M & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & c_0 & c_1 & \cdots & c_M \end{bmatrix} = [\mathbf{c}_0 \mathbf{c}_1 \cdots \mathbf{c}_L]$$

Minimum Mean Squared Error Equalization

- The equalizer output y_i is the estimate of the transmitted symbol $s_{i-\tau}$, where $0 \leq \tau \leq L$ is the equalizer's decision delay.
- The Mean-Squared Error is

$$\begin{aligned} \text{MSE}(\mathbf{w}) &= \mathrm{E}[|s_{i-\tau} - y_i|^2] \\ &= \mathrm{E}[(s_{i-\tau} - y_i)(s^*_{i-\tau} - y^*_i)] \\ &= \underbrace{\mathrm{E}[s_{i-\tau} s^*_{i-\tau}]}_{\sigma_s^2} - \mathrm{E}[s^*_{i-\tau} y_i] - \mathrm{E}[s_{i-\tau} y^*_i] + \mathrm{E}[y_i y^*_i] \end{aligned}$$

$$\mathrm{E}[s^*_{i-\tau} y_i] = \mathrm{E}[s^*_{i-\tau} \mathbf{w}^H (\mathbf{C}\mathbf{s}_i + \mathbf{n}_i)] = \sigma_s^2 \mathbf{w}^H \mathbf{c}_\tau$$

$$\mathrm{E}[s_{i-\tau} y^*_i] = \sigma_s^2 \mathbf{w}^T \mathbf{c}^*_\tau$$

$$\mathrm{E}[y_i y^*_i] = \sigma_s^2 \mathbf{w}^H \mathbf{C}\mathbf{C}^H \mathbf{w} + \sigma_n^2 \mathbf{w}^H \mathbf{w} = \sigma_s^2 \mathbf{w}^H \left(\mathbf{C}\mathbf{C}^H + \frac{\sigma_n^2}{\sigma_s^2} \mathbf{I}_{M+1} \right) \mathbf{w}$$

Minimum Mean Squared Error Estimation

- The optimal MMSE solution is $\mathbf{w}_0$ that minimizes the MSE

$$\begin{aligned}\mathbf{w}_0 &= \arg\min_{\mathbf{w}} \text{MSE}(\mathbf{w}) \\ &= \arg\min_{\mathbf{w}} \sigma_s^2 \left(1 - \mathbf{w}^H \mathbf{c}_\tau - \mathbf{w}^T \mathbf{c}_\tau^* + \mathbf{w}^H \left(\mathbf{C}\mathbf{C}^H + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}\right)\mathbf{w}\right)\end{aligned}$$

- The MMSE solution is obtained by setting the gradient vector of MSE to zero

$$\nabla_{\mathbf{w}} \text{MSE}(\mathbf{w}) = \mathbf{0}$$

Minimum Mean Squared Error Estimation

- The gradient vector of MSE is

$$\nabla_{\mathbf{w}} \text{MSE}(\mathbf{w}) = -\mathbf{c}_\tau + \left(\mathbf{C}\mathbf{C}^H + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}\right)\mathbf{w}$$

- Therefore,

$$-\mathbf{c}_\tau + \left(\mathbf{C}\mathbf{C}^H + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}\right)\mathbf{w}_0 = \mathbf{0}$$

- The MMSE equalizer weights are

$$\boxed{\mathbf{w}_0 = \left(\mathbf{C}\mathbf{C}^H + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}\right)^{-1} \mathbf{c}_\tau}$$